

(m,n)-type holographic dark energy models

Yi Ling ^{1,2*} and Wen-Jian Pan ^{2,1†}

¹*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*

² *Center for Relativistic Astrophysics and High Energy Physics,
Department of Physics, Nanchang University, 330031, China*

Abstract

We construct (m, n) -type holographic dark energy models at a phenomenological level, which can be viewed as a generalization of agegraphic models with the conformal-like age as the holographic characteristic size. For some values of (m, n) the holographic dark energy can automatically evolve across $\omega = -1$ into a phantom phase even without introducing an interaction between the dark energy and background matter. Our construction is also applicable to the holographic dark energy with generalized future event horizon as the characteristic size.

*Electronic address: lingy@ihep.ac.cn

†Electronic address: wjpan@zhgkxy@163.com

I. INTRODUCTION

Recent cosmological observations have disclosed the current accelerated expansion of the universe driven by the exotic energy with negative pressure, which is dubbed as dark energy(DE)[1–4]. The dark energy scenario has attracted a great deal of attention in the last decade. Despite of many efforts in this subject, the nature of DE is the most mysterious problem in modern cosmology. The simplest candidate of dark energy is Λ CDM model, in which $\omega = -1$ is constant. Although being consistent with all observations very well, this model undergoes the fine-tuning problem and the coincidence problem[5, 6]. After this, a lot of dynamical DE models have been proposed to solve these problems (for recent reviews we refer to [7, 8]). As a matter of fact, for any dynamical dark energy model it contains a degree of freedom to specify, namely the state parameter ω in the equation of state, which in first principle should be derived at a statistical level, like what we have done for the ordinary matter ($\omega_m = 0$) and radiation ($\omega_r = \frac{1}{3}$). Unfortunately we know little about the microscopic property of dark energy such that the statistical mechanics on dark energy is missing. As a result, one needs to look for some principle to govern the dynamics of the state parameter of dark energy such that the evolution of the universe can be uniquely determined. Recently the most popular strategy is probably applying the holographic principle[9–12]. Motivated by this principle one proposed that in a cosmological setting the total energy of system with size L should not exceed the mass of a black hole with the same radius, namely

$$L^3 \Lambda^4 = L^3 \rho_\Lambda \leq LM_p^2. \quad (1)$$

While saturating this inequality by choosing the largest L it gives rise to a holographic energy density

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}, \quad (2)$$

where c is a dimensionless constant. One usually call the dark energy satisfying equation (2) as holographic dark energy. Now the key issue is how to choose the holographic characteristic scale L . During the past years, there are many possible choices in the literature[13–23], in which the holographic scale L can be identified with the future event horizon[18], the conformal age of the universe[20] or the Ricci scalar of the universe[21]. Specially the models taking the conformal age of the universe are also dubbed as new agegraphic dark energy models. Recently Huang and Wu proposed a new holographic dark energy model with a

conformal-like age of the universe as the scale L in [22]. This model can be consistent with the history from the inflation to the current universe.

Motivated by above progress in this paper we intend to propose a new type of holographic dark energy models at a phenomenological level which is characterized by two numbers (m, n) . We will demonstrate that it is quite general to construct a holographic dark energy model with an age-like scale as the holographic scale L . In particular, when (m, n) take some specific numbers all the agegraphic-like dark energy models previously proposed in the literature can be recovered. We will investigate the general features of (m, n) -type holographic dark energy models in this paper. Our paper is organized as follows. In Section 2, we construct the (m, n) type holographic dark energy model with an age-like scale as the characteristic size and discuss its general properties during the various epoches of the universe. The interaction between the dark energy and the dark (background) matter (DM) is discussed in Section 3 and the coincidence problem is addressed. In section 4 we briefly remark that our construction is also applicable to the models with generalized future event horizon as the holographic size in the same spirit. Our conclusions and discussions are given in section 5.

II. (m, n) -TYPE HOLOGRAPHIC DARK ENERGY MODELS

We start with the standard Friedmann equations in which DE and background constituent with constant of state ω_i , are assumed to be independent, without interaction between them.

$$3M_p^2 H^2 = \rho_\Lambda + \rho_i, \quad (3)$$

$$\dot{\rho}_\Lambda + 3H\rho_\Lambda(1 + \omega) = 0, \quad (4)$$

$$\dot{\rho}_i + 3H\rho_i(1 + \omega_i) = 0, \quad (5)$$

where $H = \dot{a}/a$ is the Hubble factor. In particular, $\omega_i = 0$ for pressureless matter, whereas $\omega_i = 1/3$ for radiation. For convenience through this paper we denote the ratio ρ_i/ρ_Λ by r which is related to $\Omega_\Lambda = \rho_\Lambda/(3M_p^2 H^2)$ by $1 + r = 1/\Omega_\Lambda$. From the Friedmann equation we easily obtain a relation between the characteristic size L and the Hubble factor H as

$$LH = \sqrt{1 + rc}. \quad (6)$$

Furthermore, from the equations of conservation we have

$$r' = 3(\omega - \omega_i)r, \quad (7)$$

where $r' \equiv \dot{r}/H = dr/d\ln a$. From Eqs.(3), (4) and (5) we can work out

$$\dot{H} = -\frac{3}{2}\left(1 + \frac{\omega_i r + \omega}{1+r}\right)H^2. \quad (8)$$

From Eqs.(6) and (8) one can find

$$2\frac{L'}{L} = 3\left(1 + \frac{\omega + \omega_i r}{1+r}\right) + \frac{r'}{1+r} = 3(1 + \omega). \quad (9)$$

We point out that the relations derived above are general and independent of the specific form of the holographic characteristic scale. Now we intend to construct a (m, n) -type holographic dark energy model, in which the characteristic scale L is proposed to be

$$L = \frac{1}{a^m(t)} \int_0^t a^n(t') dt', \quad (10)$$

with (m, n) being a couple of real numbers (at phenomenological level they need not be integers.). Taking the derivative with respect to $\ln a$ on both sides of the equation, we find

$$\frac{L'}{L} = -m + \frac{a^{n-m}}{HL}. \quad (11)$$

This relation together with Eq.(9) leads to the equation of state for (m, n) -type holographic dark energy,

$$\omega = -1 - \frac{2}{3}m + \frac{2}{3}\frac{a^{n-m}}{HL} = -1 - \frac{2}{3}m + \frac{2}{3}\frac{a^{n-m}}{c\sqrt{1+r}}. \quad (12)$$

In the absence of the interaction between background matter and dark energy, Equations (7) and (12) govern the evolution of r and ω . Alternatively, one can rewrite the equation of motion in terms of Ω_Λ as

$$\Omega'_\Lambda = \Omega_\Lambda(1 - \Omega_\Lambda)\left(3 + 3\omega_i + 2m - \frac{2\sqrt{\Omega_\Lambda}a^{n-m}}{c}\right). \quad (13)$$

Next we intend to figure out some basic constraints on the values of (m, n) through the investigation on the general properties of (m, n) -type holographic dark energy during the different epochs of the universe.

A. Radiation- or Matter-dominated epoch ($a \rightarrow 0$)

For a radiation-dominated or matter-dominated epoch, we find the Friedmann equation Eq.(3) can be approximately written as,

$$\rho_i \propto H^2 = A^2 a^{-3(1+\omega_i)}, \quad (14)$$

where A is a constant and ω_i is the state parameter, specifically, $\omega_i = 1/3$ for radiation and 0 for matter. This equation implies that the scale factor evolves as $a \propto t^{\frac{2}{3(1+\omega_i)}}$. Thus the holographic scale L can be explicitly integrated out as

$$L = \frac{1}{A[n + \frac{3}{2}(1 + \omega_i)]} a^{n-m+\frac{3}{2}(1+\omega_i)}. \quad (15)$$

This solution leads to an important relation, implying that the ratio appearing in Eq.(12) approaches to a constant during the radiation-dominated or matter-dominated epoch.

$$\frac{2a^{n-m}}{3HL} = \frac{2}{3}n + 1 + \omega_i. \quad (16)$$

As a result, one can easily find that Ω_Λ during that epoch evolves as

$$\Omega_\Lambda = (n + \frac{3}{2} + \frac{3\omega_i}{2})^2 c^2 a^{2m-2n}. \quad (17)$$

It is easy to check that the above equation is consistent with Eq.(13). Moreover, the state parameter of dark energy is going to a constant, which is

$$\omega = \frac{2}{3}(n - m) + \omega_i. \quad (18)$$

Obviously, the state parameter depends on the values of (m, n) . We have the following remarks on the constraints on the values of (m, n) .

- If $n > m$, then $\omega > \omega_i \geq 0$. It means r will increase with the expansion of the universe such that the universe could never exit from a radiation-dominated or matter-dominated epoch. Thus this case is ruled out and we will not consider it in next sections.
- If $n < m$, then $\omega < \omega_i$ during the Radiation- or matter-dominated epoch. In particular, when $n - m = -1$, we have $\omega = -\frac{2}{3}$ and $r \propto a^{-2}$ for $\omega_i = 0$, while $\omega = -\frac{1}{3}$ and $r \propto a^{-2}$ for $\omega_i = 1/3$. This situation recovers the new agegraphic dark energy model [20] which

is $(m, n) = (0, -1)$, and the conformal age-like holographic dark energy model which is $(m, n) = (4, 3)$ [22]. In addition, we notice that in this case as $a \rightarrow 0$, the ratio r goes to infinity such that there is no constraint on the value of the constant c .

- If $m = n$, then $\omega = \omega_i$ and $\rho_\Lambda \propto \rho_i$. This situation is very subtle and previously a similar discussion has been presented for the old agegraphic dark energy model which corresponds to the special case with $(m, n) = (0, 0)$ [19]. Since $\omega = \omega_i$, the ratio between the dark energy and dark matter/radiation would be a constant

$$r = \frac{1}{c^2[n + \frac{3}{2}(1 + \omega_i)]^2} - 1 > 0. \quad (19)$$

For $\omega_i = 1/3$, its positivity requires

$$c < \frac{1}{m + 2}. \quad (20)$$

$\omega = \omega_i$ implies that dark energy might intend to track the behavior of the dominated ingredient in the early stage of the universe, and thus might have the same origin with dark matter. This potential possibility of unifying dark matter and dark energy is very interesting. However, to implement this scenario one need introduce a mechanism to make dark energy deviate from dark matter and finally the impact of dark energy must be large enough to be responsible for the acceleration of the universe at late times. We remark that in the absence of such a mechanism this scenario is hard to be realized. This difficulty might be overcome by introducing a suitable interaction between dark energy and dark matter, but here we leave this issue for further investigation in future.

B. Future with $a \gg 1$

When the interaction between dark energy and dark matter is not taken into account we only consider the case of $n < m$. When $a \gg 1$, the asymptotic behavior of the universe in future will be described by the following equations

$$\omega = -1 - \frac{2}{3}m, \quad (21)$$

$$r' = -(3 + 2m)r. \quad (22)$$

First of all, if $m \leq -1$, then $\omega > -1/3$. It means that the universe will not stay in an accelerating phase for ever. An example discussed in the previous literature is taking the particle horizon as the holographic characteristic scale, which corresponds to $m = n = -1$.

- If $m > 0$, then the holographic dark energy will behave like a phantom field.
- If $m = 0$, then the holographic dark energy will approach a cosmological constant. The specific example is the new agegraphic dark energy models with $n = -1, m = 0$.
- If $-1 < m < 0$, the holographic dark energy can drive the universe into an accelerating phase indeed. The key point is whether this choice will be consistent with our observational data about the present universe.

C. Present days

The most strict constraints come from the observation data on our present universe. Here we only roughly estimate the possible values for m and c . First of all, our current universe has an accelerating expansion, which requires that

$$1 + r_0 + 3\omega_0 < 0. \quad (23)$$

We find it leads to

$$m > -1 + \frac{r_0}{2} + \frac{1}{c\sqrt{1+r_0}}. \quad (24)$$

Since c is a positive number, if we plug the current $r_0 \simeq \frac{1}{3}$ into this inequality, then we find a bound for m , which is

$$m > -\frac{5}{6}. \quad (25)$$

Conversely, for a given m , we find the constant c is constrained by

$$c > \frac{3\sqrt{3}}{6m+5}. \quad (26)$$

If we further require $\omega_0 \simeq -1$, c can be uniquely fixed by equation, which is $c = 1/(m\sqrt{1+r_0})$ ($m > 0$ only). In particular, when $(m, n) = (0, -1)$ and $(m, n) = (4, 3)$, our above estimation is in a good agreement with the results obtained by more severe constraints from observation data[24, 25]. This implies that other types such as $(m, n) = (1, 0), (2, 1), (3, 2)$ can also fit the data very well.

As a summary, we find the basic constraints on the (m, n) -type holographic dark energy are $n < m$ and $m > -5/6$.

III. HOLOGRAPHIC DARK ENERGY WITH INTERACTION

Although the nature of both DM and DE still remains a mystery, the possibility that DE and DM can interact with each other has been widely discussed recently [26–37]. Moreover, observational signatures on the interaction between dark ones have been investigated in the probes of the cosmic expansion history with the use of the SNIa, BAO and CMB shift data [38–40]. The interacting dark energy has also been considered as a possible solution to the coincidence problem [26, 41–50].

In this section we intend to extend (m, n) -type holographic dark energy models with interactions. When the interaction is taken into account, the equations of motion for ρ_Λ and ρ_i become

$$\dot{\rho}_\Lambda = -3H\rho_\Lambda(1 + \omega) - Q, \quad (27)$$

$$\dot{\rho}_i = -3H\rho_i(1 + \omega_i) + Q, \quad (28)$$

where Q denotes the interacting term. From (27) and (28) we find that the interacting term has the following general form,

$$\tilde{Q} \equiv \frac{Q}{H\rho_\Lambda} = \frac{1}{1+r}[r' - 3(\omega - \omega_i)r]. \quad (29)$$

It is also easy to derive a general relation between L and \tilde{Q} as

$$\tilde{Q} = r' - 2r\left(\frac{L'}{L} - \frac{3}{2} - \frac{3\omega_i}{2}\right). \quad (30)$$

As we stressed in Ref.[26], four free parameters ω, r, L and Q are not independent. Given any two of them, the dynamics of the other two will be determined. Usually, people propose the forms of L and Q , and then find out the evolutions of ω and r with observation data. Thus, after introducing the interacting term we find the equations for ω and r in the previous section can be generalized as

$$\omega = -1 - \frac{2}{3}m + \frac{2}{3} \frac{a^{n-m}}{c\sqrt{1+r}} - \frac{\tilde{Q}}{3}. \quad (31)$$

$$r' = \tilde{Q}(1+r) + 3(\omega - \omega_i)r. \quad (32)$$

Obviously the interaction will change the dynamics of ω as well as r . One can alternatively write down the equation of motion for Ω_Λ as

$$\Omega'_\Lambda = \Omega_\Lambda \left[(3 + 3\omega_i + 2m - \frac{2a^{n-m}\sqrt{\Omega_\Lambda}}{c})(1 - \Omega_\Lambda) - \tilde{Q}\Omega_\Lambda \right]. \quad (33)$$

It is clear that Eq.(31) and Eq.(33) reduce to Eq.(12) and Eq.(13) respectively in the case of $\tilde{Q} = 0$. Now we turn to consider the coincidence problem with the help of interaction. We expect that the ratio r of dark matter to dark energy density varies slowly, and will finally approach to a non-zero constant at late time. For explicitness, we consider a specific form of the interaction $\tilde{Q} = 3b^2(r + 1)$, where b^2 is a coupling constant. Its positivity is responsible for the transition from dark energy to dark matter. Repeating the calculations in the previous section, we find that the basic constraint on m and c becomes

$$m > -\frac{5}{6} - 2b^2, \quad (34)$$

$$c > \frac{3\sqrt{3}}{6m + 5 + 12b^2}. \quad (35)$$

To alleviate the coincidence problem, we are more concerned with the asymptotic value of r as ($a \gg 1$). Setting $r' = 0$ in Eq.(32), we find the ratio of dark matter to dark energy will approach to a non-zero constant, which is

$$r_f = \frac{3b^2}{3 + 2m - 3b^2}, \quad (36)$$

where the value of r_f depends on m and b manifestly. This result indicates that if m is not too large, the situation that the ratio r keeps staying in a region with unit order can be easily realized, thus providing a mechanism to understand the coincidence problem.

IV. (m, n) TYPE MODELS WITH A GENERALIZED FUTURE EVENT HORIZON

In this section, we would like to point out that with the same spirit our construction should be applicable to the holographic dark energy models with generalized future event horizon as the characteristic size, which has been extensively studied in literature[8]. Explicitly, we may generalize the definition of the holographic characteristic scale to

$$L = \frac{1}{a^m(t)} \int_t^\infty a^n(t') dt'. \quad (37)$$

Specially, when $(m, n) = (-1, -1)$ it recovers the ordinary holographic dark energy models with future event horizon. In this definition taking the derivative with respect to $\ln a$ on both sides, we obtain

$$\frac{L'}{L} = -m - \frac{a^{n-m}}{HL}. \quad (38)$$

With the same algebra we may derive the equation of state as

$$\omega = -1 - \frac{2}{3}m - \frac{2a^{n-m}\sqrt{\Omega_\Lambda}}{3c}, \quad (39)$$

while the equation of motion for Ω_Λ reads as

$$\Omega'_\Lambda = \Omega_\Lambda(1 - \Omega_\Lambda)(3 + 3\omega_i + 2m + \frac{2a^{n-m}\sqrt{\Omega_\Lambda}}{c}). \quad (40)$$

In general case without interaction we still require that $n \leq m$ such that the proportion of dark energy always increases with the evolution of the universe if $m > -\frac{3}{2}$. Moreover, under the condition of acceleration $1 + r + 3\omega < 0$, and with the use of $r_0 \simeq 1/3$ we find the number m should be subject to the inequality

$$m > -\frac{5}{6} - \frac{\sqrt{3}}{2c}. \quad (41)$$

It is interesting to notice that the fate of the universe would be very different for $n = m$ and $n < m$ in future with $a \gg 1$. For $n < m$, we easily find that the asymptotic behavior of the state parameter will be depicted by the equation

$$\omega = -1 - \frac{2}{3}m, \quad (42)$$

while for $m = n$, we find its value will approach to

$$\omega = -1 - \frac{2}{3}m - \frac{2}{3c}, \quad (43)$$

which depends on the constant c . The latter ones of course cover the ordinary holographic dark energy model with $m = n = -1$ and $c = 1$. Based on our construction we could consider a generalized model with $m = -1$ and $n = -1 - \delta$ where δ is a small positive constant. From our above consideration it is expected that this modification will change the asymptotical behavior of the dark energy dramatically. From this point our construction here is quite different from the generalized holographic models with varying $c(z)$, which has recently been proposed in [51].

V. DISCUSSION AND CONCLUSIONS

In this paper we have constructed (m, n) type holographic dark energy model which can be viewed as a generalization of the ordinary holographic dark energy models appeared in

literature. At phenomenological level, such a generalization provides us more space in theory to fit the observational data. In particular, for some specific values of (m, n) the equation of state ω can naturally evolve cross phantom divide in the entire evolution of the universe even without introducing an interaction between dark energy and background matter. We have discussed the general features of age-like holographic dark energy models in various epoches of the universe and derived the basic constraints on the values of (m, n) . We have also remarked that this construction is applicable to the holographic models with generalized future event horizon as the characteristic scale.

For age-like holographic models, the case of $m = n$ is special. In this case it seems that dark energy has the same behavior as the dominant ingredient in the early epoches of the universe, implying that dark energy might be unified with dark matter, analogous to what happened in cosmological models with generalized Chaplygin gas[52]. However, if DE and DM were unified at early stages, we must introduce some mechanism to make dark energy deviate from dark matter state, and eventually become dominant to be responsible for the acceleration of the universe. This might be implemented by introducing some appropriate interactions between dark energy and dark matter, and our investigation is under progress.

A key issue on this type of holographic dark energy models is the stability under perturbations. Previously some relevant discussions on ordinary holographic dark energy models have been presented in previous literature [53–55], and we expect to investigate this issue in near future.

Note: After we uploaded our manuscript, the observational constraint on this model appeared in [56], in which (m, n) are taken as integers, such as $(m, n)=(1, 0), (2, 1)$ and $(3, 2)$. Their analysis indicates that our model is consistent with the observational data very well.

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